Does the Deuce Scoring System in Tennis benefit the better player, and by how much?

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1 Introduction

Tennis has always been a big part of my life. I am a four-year varsity athlete and have attended tennis IASAS (Interscholastic tournament) three times. At IASAS, the scoring differs greatly from conventional tennis scoring. Rather than a best of five sets, just one "superset" is played, where the first to win eight total games is victorious. Furthermore, the deuce-system has been removed at IASAS to speed up play. As a player, certain intuitive feelings are present on the court. When playing against a better player, I feel as though deuce is to my disadvantage, however, against a less-skilled player, a deuce feels as though it adds to my advantage. This is based on no fact, as of yet, just a gut feeling. Knowing the point probabilities, I wish to explore the mathematical relationship between a player's skill level, and the advantage/disadvantage different scoring mechanisms can provide and test whether this intuitive sensation, is in fact based in mathematics.

1.1 Scoring System

Tennis is unusual in the way that points are accumulated in that the scoring system differs greatly from traditional ones. Vocabulary will need to be defined, as though to not cause confusion throughout the investigation.

1.1.1 Key Vocabulary

- **Games**: The winning of points in the fashion: zero, fifteen (one point), thirty (two points), forty (three points), game (four points). To win the game, the player must win the four points before the other. In the case of a forty-all tie (three points each), a deuce is played out (the game will not end until two consecutive points are won) At a deuce, when a point is won, it is called the "advantage".
- **Set**: The winning of six games, or in the case of a five-all tie, the further winning of two consecutive games, or in the case of a six-all tie, the winning of a 7 point tiebreak (first to seven points).
- **Match**: The winning of three sets (men's tennis)

1.1.2 Additional Information

- Points are exchanged; Points can be for or against a player
- After each game, service is exchanged
- In the case of a two-all tie in sets, the fifth set does not have a tiebreak at a six-all game score tie, but rather continues until a player wins two more consecutive games.

1.2 Tennis Scoring Origin

Many wonder why the scoring of tennis is at it is, and it has to do with the long history of the sport. Before technological advancements, tennis was scored on a clock face. The clock was divided into quarters, thus requiring four points to win a game (reach the sixty). It is, therefore, clear why one point should equal 15 and two points equals thirty. However, why is three points equal to forty, instead of forty-five? This is simply because of the pronunciation of tennis scoring. Forty-five and its three syllables ruined the rhythm when inline with the other double-syllable scores. For this reason, forty-five was truncated to forty overtime.

2 No-Deuce Scoring: Best of Five Sets

First, I will investigate the probabilities of winning a full tennis match with a no-deuce system (sudden death). For this, sample point probabilities (probability of the player winning an individual point) will have to be taken.

2.1 Initial Assumptions

Initial assumptions will have to be made throughout the exploration in order to simplify the mathematics and provide more reasonable results.

- The probability of a player winning the point is independent - does not rely on the outcome of previous points.
- The point probability remains constant throughout the match regardless of service, weather or any other external factors.
- There is no rule of the fifth set having no tiebreak (it is assumed a tiebreak is played in the fifth set).

Given Point Probability of **Adam** $P_A = 0.6$
Given Point Probability of **Brian** $P_B = 0.4$

The point probabilities were chosen as such to:

1. Represent the difference in skill level(better player vs worse player)
2. Simplify the calculations, notations, and results (variables are confusing)

2.2 Game Probability: Adam

The combinations for **Adam** winning a game are as follows, where A still represents **Adam** winning a point.

$AAAA$	**Adam** wins four consecutive points
$AAABA$	**Adam** loses one point but wins the game
$AAABBA$	**Adam** loses two points but wins the game
$AAABBBA$	**Adam** loses three points but wins the game

However, these are only the combinations, and to get the true probability they must become permutations with some alike elements since the arrangement in which the points are won is important to the context of the investigation. **Adam** has to win the last point, in order to win the game. Since PA equals 0.6, the resulting probabilities for each arrangement are the combination of the number of points won by **Adam**, times the number of ways these points can be arranged times the final point that **Adam** must win. They are listed as follows:

$AAAA$	$(0.6)^4$
$AAABA$	$(C(4,1) \times (0.6)^3 \times (0.4)^1) \times (0.6)$
$AAABBA$	$(C(5,2) \times (0.6)^3 \times (0.4)^2) \times (0.6)$
$AAABBBA$	$(C(6,3) \times (0.6)^3 \times (0.4)^3) \times (0.6)$

With this, the total probability of **Adam** winning a game (P_{A_G}) is the sum of the probabilities shown above.

$$P_{A_G} = (0.6)^4 + (C(4,1) \times (0.6)^3 \times (0.4)^1) \times (0.6) + (C(5,2) \times (0.6)^3 \times (0.4)^2) \times (0.6) + (C(6,3) \times (0.6)^3 \times (0.4)^3) \times (0.6)$$

$$P_{A_G} \approx 0.710$$

The answer was truncated to make results easier to understand and maintain coherence throughout the exploration.

2.3 Game Probability: Brian

Similarly, the combinations for **Brian** winning a game are as follows:

$BBBB$	**Brian** wins four consecutive points
$BBBAB$	**Brian** loses one point but wins the game
$BBBAAB$	**Brian** loses two points but wins the game
$BBBAAAB$	**Brian** loses three points but wins the game

The corresponding probabilities, therefore, are as such:

$BBBB$	$(0.4)^4$
$BBBAB$	$(C(4,1) \times (0.4)^3 \times (0.6)^1) \times (0.4)$
$BBBAAB$	$(C(5,2) \times (0.4)^3 \times (0.6)^2) \times (0.4)$
$BBBAAAB$	$(C(6,3) \times (0.4)^3 \times (0.6)^3) \times (0.4)$

With this, the total probability of **Brian** winning a game P_{B_G} is the sum of the probabilities shown above.

$$P_{B_G} = (0.4)^4 + (C(4,1) \times (0.4)^3 \times (0.6)^1) \times (0.4) + (C(5,2) \times (0.4)^3 \times (0.6)^2) \times (0.4) + (C(6,3) \times (0.4)^3 \times (0.6)^3) \times (0.4)$$

$$P_{B_G} \approx 0.290$$

We can check this value, as the probability of **Brian** winning a game and the probability of **Adam** winning a game should sum up to 1.

$$P_{A_G} + P_{B_G} = 1 \implies 0.710 + 0.290 = 1 \checkmark$$

To win a set, a player must possess six games, or in the case of a five-all draw, seven games. In the case of a six-all draw, a tiebreak will commence.

In the future, only **Adam**'s probabilities will be explicitly calculated, and **Brian**'s will just be calculated by subtracting **Adam**'s from 1.

2.4 Set Probability:Adam

Listed below are the combinations of games that will win **Adam** a set.

$AAAAAA$	**Adam** wins six games and wins the set
$AAAAABA$	**Adam** loses one game but wins the set
$AAAAABBA$	**Adam** loses two games but wins the set
$AAAAABBBA$	**Adam** loses three games but wins the set
$AAAAABBBBA$	**Adam** loses four games but wins the set
$AAAAABBBBB$	**Adam** loses 5 games (5-5) draw
$\|AB$	6-6 draw - tiebreak
$\|BA$	6-6 draw - tiebreak
$\|AA$	**Adam** wins the set
$\|BB$	**Adam** loses the set

The tiebreak scenario will be temporarily left, as although there is still a possibility that**Adam** wins the set through a tiebreak, it will be added into the equation later. The probabilities for the aforementioned combinations are listed below.

$AAAAAA$	$(0.710)^6$
$AAAAABA$	$(C(6,1) \times (0.71)^5 \times (0.29)^1) \times (0.71)$
$AAAAABBA$	$(C(7,2) \times (0.71)^5 \times (0.29)^2) \times (0.71)$
$AAAAABBBA$	$(C(8,3) \times (0.71)^5 \times (0.29)^3) \times (0.71)$
$AAAAABBBBA$	$(C(9,4) \times (0.71)^5 \times (0.29)^4) \times (0.71)$
$AAAAABBBBB$	$(C(10,5) \times (0.71)^5 \times (0.29)^5)$
$\|AB$	6-6 draw - tiebreak
$\|BA$	6-6 draw - tiebreak
$\|AA$	**Adam** wins the set
$\|BB$	**Adam** loses the set

The probability of **Adam** winning a set before the five-all possibility is:

$$\begin{aligned} P_{A_S} = (0.710)^6 + (C(6,1) \times (0.71)^5 \times (0.29)^1) \times (0.71) + (C(7,2) \times (0.71)^5 \times (0.29)^2) \times (0.71) \\ + (C(8,3) \times (0.71)^5 \times (0.29)^3) \times (0.71) + (C(9,4) \times (0.71)^5 \times (0.29)^4) \times (0.71) \\ P_{A_S} \approx 0.866 \end{aligned}$$

In the last scenario, at a five-all draw, there are then four more possibilities (AA, AB, BA, BB). (AB and BA) lead to a tiebreak. Therefore, it is only the (AA) possibility that will be considered now. For**Adam** to win the set without the tiebreak, only the AA possibility works. This means the probability of him winning the set without a tiebreak is as follows:

$$\begin{aligned} P_{A_S} &= 0.866 + (C(10,5) \times (0.71)^5 \times (0.29)^5) \times (0.71)^2 \\ &\approx 0.913 \end{aligned}$$

2.4.1 Set probability with tiebreak:Adam

A tiebreak will occur if the next two games had the progression (AB or BA). A player wins a tiebreak when they reach seven points, however in the case of a six-all tie the tiebreak continues until a player has won two points consecutively. The combinations of a tiebreak win for **Adam** are as follows:

$AAAAAAA$		**Adam** wins seven points, and the tiebreak
$AAAAAABA$		**Adam** loses one point, but wins the tiebreak
$AAAAAABBA$		**Adam** loses two points, but wins the tiebreak
$AAAAAABBBA$		**Adam** loses three points, but wins the tiebreak
$AAAAAABBBBA$		**Adam** loses four points, but wins the tiebreak
$AAAAAABBBBBA$		**Adam** loses five points, but wins the tiebreak
$AAAAAABBBBBB$		**Adam** loses six points, six-all draw
	$\|AB$	**Adam** wins then **Brian** Wins (7-7)
	$\|BA$	**Brian** wins then **Adam** Wins (7-7)
	$\|AA$	**Adam** wins the two points and wins the tiebreak
	$\|AB$	**Adam** loses the two points and loses the tiebreak

The corresponding probabilities are as follows

$AAAAAAA$	$(0.6)^7$
$AAAAAABA$	$(C(7,1) \times (0.6)^6 \times (0.4)^1) \times (0.6)$
$AAAAAABBA$	$(C(8,2) \times (0.6)^6 \times (0.4)^2) \times (0.6)$
$AAAAAABBBA$	$(C(9,3) \times (0.6)^6 \times (0.4)^3) \times (0.6)$
$AAAAAABBBBA$	$(C(10,4) \times (0.6)^6 \times (0.4)^4) \times (0.6)$
$AAAAAABBBBBA$	$(C(11,5) \times (0.6)^6 \times (0.4)^5) \times (0.6)$
$AAAAAABBBBBB$	$(C(12,6) \times (0.6)^6 \times (0.4)^6)$
$\|AB$	$(0.6) \times (0.4)$
$\|BA$	$(0.4) \times (0.6)$
$\|AA$	$(0.6) \times (0.6)$
$\|AB$	$(0.4) \times (0.4)$

Therefore, the probability that **Adam** wins the tiebreak (before reaching the six-all tie) is

$$\begin{aligned} P_{A_T} = (0.6)^7 &+ (C(7,1) \times (0.6)^6 \times (0.4)^1) \times (0.6) + (C(8,2) \times (0.6)^6 \times (0.4)^2) \times (0.6) \\ &+ (C(9,3) \times (0.6)^6 \times (0.4)^3) \times (0.6) + (C(10,4) \times (0.6)^6 \times (0.4)^4) \times (0.6) \\ &+ (C(11,5) \times (0.6)^6 \times (0.4)^5) \times (0.6) \end{aligned}$$

$$P_{A_T} \approx 0.683$$

If there is a six-all draw, then the tiebreak won't end until a player has two points over the other. This only occurs in the scenarios (AA, and BB). Since scenarios (AB and BA) may go on continuously, until two points are won consecutively, this takes the form of an infinite geometric series.

Immediate Victory (8-6)	1 return to tie (9-7)	2 returns to tie (10-8)
AA	$(AB)(AA)$	$(AB)(AB)(AA)$
	$(BA)(AA)$	$(AB)(BA)(AA)$
		$(BA)(AB)(AA)$
		$(BA)(BA)(AA)$

Since $AB = BA = (0.6) \times (0.4) = 0.24$, the corresponding probabilities are:

No returns to deuce	1 return to deuce	2 returns to deuce
$(0.6)^2$	$(0.24)(0.6)^2$ $(0.24)(0.6)^2$	$(0.24)^2(0.6)^2$ $(0.24)^2(0.6)^2$ $(0.24)^2(0.6)^2$ $(0.24)^2(0.6)^2$
2^0 possibilities	2^1 possibilities	2^2 possibilities
$(0.6)^2$	$2(0.24)(0.6)^2$	$4(0.24)^2(0.6)^2$

Each additional number of times to which the tie in the tiebreak is returned is another additional power of 2, along with an extra factor of (0.24), therefore, 2(0.24) is the ratio to this infinite geometric series.

Hence the convergent sum is:

$$P_{TIE} = \frac{U_1}{1-r} = \frac{(0.6)^2}{1-(0.48)} \approx 0.692$$

Therefore,

$$\begin{aligned} P_{A_T} &\approx 0.683 + C(12,6) \times (0.6)^6 \times (0.4)^6 \times (0.692) \\ &\approx 0.805 \end{aligned}$$

Since the tiebreak scenario happens with the possibilities (AB and BA), the probability of a tiebreak happening must also be factored in. This happens in the possibilities (AB) and (BA), hence the probability that this happens is $(0.71)(0.29) + (0.29)(0.71) \approx 0.41$

Now, all subsequent parts have been calculated. Therefore, the final probability of **Adam** winning the set P_{A_S} can be found.

$$\begin{aligned} P_{A_S} &= 0.913 + C(10,5) \times (0.71)^5 \times (0.29)^5) \times 0.41 \times 0.805 \\ &\approx 0.944 \end{aligned}$$

2.5 Set probability with tiebreak: Brian

With the set probability without deuce, but with tiebreak being calculated for **Adam**, for **Brian**, the probability will be:

$$\begin{aligned} P_{B_S} &= 1 - P_{A_S} \\ &= 1 - 0.944 \\ &\approx 0.056 \end{aligned}$$

Surprisingly, with only a 0.20 point probability difference, **Adam** has a whopping 0.941 probability to win the set without a deuce scoring system, compared to a measly 0.059 set probability for **Brian**

2.6 Match Probability: Adam

Now, the match probability will be looked into using the corresponding set probabilities.

Here are the combinations of sets for ADAM to win the set.

AAA	Adam wins three sets and the match
$AABA$	Adam loses one set but wins the match
$AABBA$	Adam loses two sets but wins the match

Here are the corresponding probabilities:

AAA	$(0.944)^3$
$AABA$	$(C(3,1) \times (0.944)^2 \times (0.056)^1) \times (0.944)$
$AABBA$	$C(4,2) \times (0.944)^2 \times (0.056)^2) \times (0.944)$

Therefore, the match probability for **Adam** is:

$$\begin{aligned} P_{A_M} &= (0.944)^3 + (C(3,1) \times (0.944)^2 \times (0.056)^1) \times (0.944) + C(4,2) \times (0.944)^2 \times (0.056)^2) \times (0.944) \\ &\approx 0.9983 \end{aligned}$$

2.7 Match Probability: Brian

Brian's match probability can be calculated again using **Adam's** match probability.

$$\begin{aligned} P_{B_M} &= 1 - P_{A_M} \\ &= 1 - 0.998323 \\ &\approx 0.001676 \end{aligned}$$

With only differing point probabilities of 0.4, to 0.6, the resultant match probabilities are significantly more diverse. Perhaps this has to do with the scoring mechanism of tennis, and how it strongly favours the better overall player and the more consistent player. Now, the match probabilities with deuce will be calculated.

3 Deuce Scoring: Best of Five Sets

3.1 3.1 Game Probability: Adam

Again, the same point probabilities will be used to maintain consistency.

$$\text{Given Point Probability of Adam } (P_A) = 0.6$$
$$\text{Given Point Probability of Brian } (P_B) = 0.4$$

Adam's probability of winning a game should now change, due to the deuce factor. Once again, the possible combinations will be listed below:

$AAAA$	**Adam** wins four consecutive points
$AAABA$	**Adam** loses one point but wins the game
$AAABBA$	**Adam** loses two points but wins the game
$AAABBB$	**Adam** loses three points causing a deuce
$\|AB$	**Adam** wins, then **Brian** wins : deuce
$\|BA$	**Brian** wins, then **Adam** wins : deuce
$\|AA$	**Adam** wins both points, and the game
$\|BB$	**Adam** loses both points, and the game

For the first three scenarios, the probability of Adam winning the game is unchanged, because the deuce plays no effect. However, it is in the last case where the deuce plays into the calculation. Here are the corresponding probabilities for each scenario:

$AAAA$	$(0.6)^4$
$AAABA$	$(C(4,1) \times (0.6)^3 \times (0.4)^1) \times (0.6)$
$AAABBA$	$(C(5,2) \times (0.6)^3 \times (0.4)^2) \times (0.6)$
$AAABBB$	$(C(6,3) \times (0.6)^3 \times (0.4)^3)$
$\|AB$	$(0.6) \times (0.4)$
$\|BA$	$(0.4) \times (0.6)$
$\|AA$	$(0.6) \times (0.6)$
$\|BB$	$(0.4) \times (0.4)$

Therefore the probability that **Adam** wins from the first three possibilities, same as before, is:

$$\begin{aligned} P_{A_G} &= (0.6)^4 + (C(4,1) \times (0.6)^3 \times (0.4)^1) \times (0.6) + (C(5,2) \times (0.6)^3 \times (0.4)^2) \times (0.6) \\ &\approx 0.544 \end{aligned}$$

In this case, with a deuce, the game will only end if either **Adam** or **Brian** win both consecutive points. If they both win one point (AB or BA), the game will return to deuce. This mathematically forms a geometric series. Possibilities at the 40-all tie include:

No returns to deuce	1 return to deuce	2 returns to deuce
AA	$(AB)(AA)$ $(BA)(AA)$	$(AB)(AB)(AA)$ $(AB)(BA)(AA)$ $(BA)(AB)(AA)$ $(BA)(BA)(AA)$

There is a distinct pattern of geometric nature in the number of times the game will return to deuce through an (AB) or a (BA).

Since $AB = BA = (0.6) \times (0.4) = 0.24$, the corresponding probabilities are:

No returns to deuce	1 return to deuce	2 returns to deuce
$(0.6)^2$	$(0.24)(0.6)^2$ $(0.24)(0.6)^2$	$(0.24)^2(0.6)^2$ $(0.24)^2(0.6)^2$ $(0.24)^2(0.6)^2$ $(0.24)^2(0.6)^2$
2^0 possibilities	2^1 possibilities	2^2 possibilities
$(0.6)^2$	$2(0.24)(0.6)^2$	$4(0.24)^2(0.6)^2$

Similar to before, each extra time a deuce is attained, another ratio of (2)(0.24) is multiplied to the infinite geometric series.

Hence the convergent sum is once again:

$$P_{DEUCE} = \frac{U_1}{1-r} = \frac{(0.6)^2}{1-(0.48)} \approx 0.692$$

Therefore, the last scenario can now be added into the first three scenarios previously calculated to get **Adam's** final probability of winning a game with deuce.

$$\begin{aligned} P_{A_G} &= 0.544 + (C(6,3) \times (0.6)^3 \times (0.4)^3) \times 0.692 \\ &\approx (0.735) \end{aligned}$$

3.2 Game Probability: Brian

Brian's match probability can be calculated again using **Adam's** match probability.

$$\begin{aligned} P_{B_M} &= 1 - P_{A_M} \\ &= 1 - 0.735 \\ &= 0.265 \end{aligned}$$

Already, it can be seen that the deuce favours the better player, as without deuce, Adam had a game probability of 0.710, while with the deuce he has a probability of 0.735. The difference this will make in the set and match probabilities will now be examined.

3.3 Set Probability: Adam

Similar to the calculation before, the game probability of **Adam** can be used to calculate his set winning probability. The calculation is entirely the same as in 2.4, but the game probability used is 0.735 instead of 0.710.

This results in:

$$P_{A_S} \approx 0.963$$

3.4 Set Probability: Brian

Using this value, **Brian's** set probability with the tiebreak and deuce can be calculated.

$$\begin{aligned} P_{B_S} &= 1 - P_{A_S} \\ &= 1 - 0.963 \\ &\approx 0.037 \end{aligned}$$

3.5 Match Probability: Adam

Using the same process as in 2.6, and the set probability we found in 3.3, we can find Adam's match probability with deuce to be:

$$P_{A_M} = 0.9995$$

3.6 Match Probability: Brian

$$\begin{aligned} P_{B_M} &= 1 - P_{A_M} \\ &= 1 - 0.9995 \\ &\approx 0.0005 \end{aligned}$$

4 Conclusion

Final values for match probabilities for **Adam** and **Brian** with and without deuce are as follows:

No Deuce:	**With Deuce:**
$P_{A_M} = 0.998$	$P_{A_M} = 0.9995$
$P_{B_M} = 0.002$	$P_{B_M} = 0.0005$

With this, it is clear that being a better player (with a greater point probability), a deuce increases the probability of winning the match, as 0.9995 ¿ 0.998. However, what is more surprising about the results is the sheer difference in the point probabilities and the match probabilities. It shows the importance of consistency in tennis, and increasing overall point probability, as with a small difference in the amount of points won, there is a significant difference in the probability of winning the match.

This exploration was based on key assumptions. The main one was the independence of point probabilities for the players. This of course is inaccurate in depicting real tennis, as there are numerous factors affecting the players skill level on the day of the match: weather, confidence, fans etc. However, the exploration is quite useful in comparing general skill level, and the corresponding benefit of deuce in tennis. Connecting this back to IASAS tennis, although it is not a best of five-sets, the sudden death system favours the "worse" player, and now it has been mathematically proven. Perhaps it would be interesting to explore the difference specifically in an eight-game match without deuce, as it is in IASAS, and compare it to the men's tour. In my opinion, after the results of this investigation, sudden death seems the better mechanism in the game. Not only does it save time, but also makes the game closer - giving the underdog a minutely small, but better, chance to take the game.

This exploration also prompted me to think about my own point probabilities compared with friends. Most games are relatively close - with match probabilities of 0.6 or 0.4 even if there is clearly a better player. Never did I believe I had a 0.9995 probability of winning a match against a player I am better than. This made me wonder whether the point probabilities I chose for this investigation were in fact very steep and that in the real world point probabilities actually range from 0.5 - 0.53 or somewhere in that ballpark, even against players you are confident you are far better than.

4.1 Further Exploration

After seeing these results, I wanted to see if I could see the match probabilities for different point probabilities or a general solution for any given input. To do this, I wrote a computer program to use the mathematics I have already used in this exploration, but where I could manipulate the inputted point probabilities. Below is a table showing the results from the program.

Without Deuce			
Point Probability **A**	Point Probability **B**	Match Probability **A**	Match Probability **B**
0.1	0.9	$6.63 \cdot 10^{-39}$	1.0
0.2	0.8	$1.96 \cdot 10^{-19}$	1.0
0.3	0.7	$2.67 \cdot 10^{-9}$	0.99999
0.4	0.6	$1.64 \cdot 10^{-3}$	0.99835
0.5	0.5	0.5	0.5
0.6	0.4	0.99835	$1.64 \cdot 10^{-3}$
0.7	0.3	0.99999	$2.67 \cdot 10^{-9}$
0.8	0.2	1.0	$1.96 \cdot 10^{-19}$
0.9	0.1	1.0	$6.63 \cdot 10^{-39}$

Seeing this pattern, I was intrigued to look more closely at point probabilities between 0.4 - 0.6

Without Deuce			
Point Probability **A**	Point Probability **B**	Match Probability **A**	Match Probability **B**
0.52	0.48	0.729	0.271
0.54	0.46	0.886	0.114
0.56	0.44	0.964	0.036
0.58	0.42	0.991	0.009

The rate at which the match probability grows is far greater than that of the point probabilities, which provides an interesting context to the sport of tennis and the importance of winning each and every point.

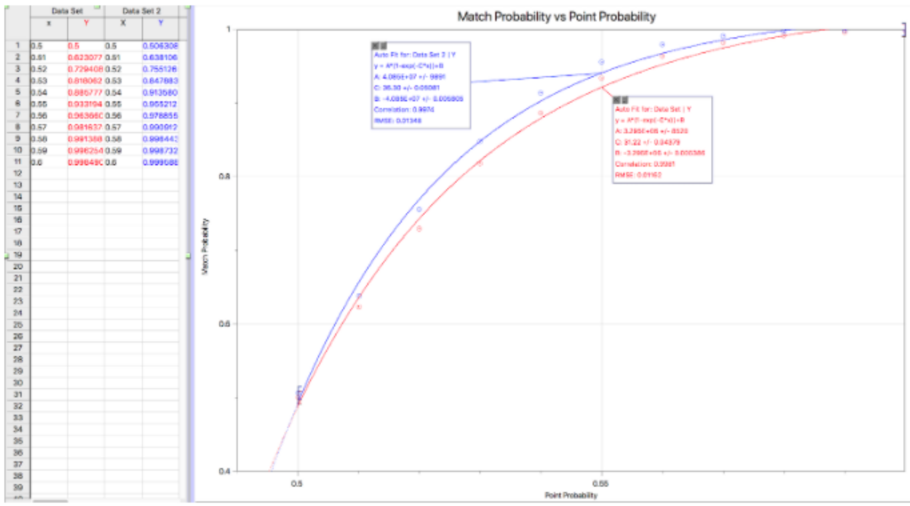
Here are the results for the “With Deuce” condition:

With Deuce			
Point Probability **A**	Point Probability **B**	Match Probability **A**	Match Probability **B**
0.1	0.9	$7.17 \cdot 10^{-44}$	1.0
0.2	0.8	$1.00 \cdot 10^{-22}$	1.0
0.3	0.7	$4.77 \cdot 10^{-11}$	0.99999
0.4	0.6	$4.73 \cdot 10^{-4}$	0.9995
0.5	0.5	0.5	0.5
0.6	0.4	0.9995	$4.73 \cdot 10^{-4}$
0.7	0.3	0.99999	$4.77 \cdot 10^{-11}$
0.8	0.2	1.0	$1.00 \cdot 10^{-22}$
0.9	0.1	1.0	$7.17 \cdot 10^{-44}$

Looking closely at the results from point probabilities between 0.4 and 0.6

With Deuce			
Point Probability **A**	Point Probability **B**	Match Probability **A**	Match Probability **B**
0.52	0.48	0.755	0.245
0.54	0.46	0.916	0.086
0.56	0.44	0.979	0.021
0.58	0.42	0.996	0.004

To better visualise the data and compare the match probabilities with and without deuce, both data sets showing results from probabilities between 0.4 and 0.6 are graphed below.



This graph best summarizes the results from the exploration. The deuce scoring system provides a slight advantage to better players, as shown by the blue line in the graph. However, what seems to be more significant is the immensely steep rate at which match probabilities grow with point probabilities ranging from 0.40.6. The match probability will from 0.6 continue to approach 1, until the point probability itself becomes 1. However, basically any player with a point probability of above 0.6, will have a nearly 100% chance of winning a tennis match.